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Purchasing power parity, unit roots, and dynamic structure

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Abstract

Recent studies of purchasing power parity (PPP) account for the possible presence of unit roots in nominal exchange rates and relative price indices by applying standard unit-root tests to real exchange rates, which are ratios of nominal exchange rates and relative price indices. These studies occasionally find evidence of PPP, but as a whole, the evidence is not definitive. Standard unit-root tests impose a restrictive dynamic structure between nominal exchange rates and relative price indices. I specify and estimate a generalized dynamic structure. I reject the dynamic restrictions implicit in standard unit-root tests of PPP, and find stronger evidence of PPP than do most other recent studies.

JEL classification: F31

Keywords: Purchasing power parity; Error correction; Unit root

1. Introduction

With arbitrage, the exchange rate between two currencies should equal the cost of purchasing a basket of goods with one currency divided by the cost of purchasing the same basket with the other currency; this arbitrage condition is known as purchasing power parity (PPP). A large number of studies test for PPP using time-series data, with early studies generally providing some support for

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PPP. [1] A shift in empirical methodology occurred following Meese and Singleton's result that nominal exchange rates apparently contain unit roots (Meese and Singleton, 1982). To accommodate unit roots, many recent studies test for PPP by testing for a unit root in the *real* exchange rate, defined to be the nominal exchange rate divided by the relative price of a basket of commodities. Under PPP deviations from a constant real exchange rate should not be permanent. Because unit-root processes have deviations that are permanent, these studies therefore take rejection of a unit root in the logarithm of the real exchange rate as evidence of PPP, and failure to reject a unit root as failure to find evidence of PPP.

Interestingly, unit-root tests as often as not fail to turn up evidence of PPP. One possible explanation, of course, is that purchasing power parity may simply not hold. (This would be striking given the large volume of international trade in commodities and currencies.) A second possible explanation is that unit-root tests generally have difficulty distinguishing unit-root processes from stationary processes with substantial persistence. This is an issue of the span of the data. With a short data span, it is difficult to reject a unit root and thereby find evidence of PPP, even if PPP holds. [2] With longer data spans there is some tendency to find evidence of PPP but even with longer data spans the evidence is not definitive. [3]

A third possible explanation, studied here, is that the unit-root tests typically used to test PPP implicitly impose a restrictive dynamic structure on the adjustment process relating nominal exchange rates and relative price indices. Two dynamic restrictions are implicit in traditional unit-root tests. First, a change in the relative price is assumed to be reflected immediately and fully in a change in the exchange rate; second, the coefficients on any lag of the relative price index and on the corresponding lag of the nominal exchange rate sum to zero. Failure to find evidence of PPP using unit-root tests on the real exchange rate may therefore simply be evidence against these two restrictions. To test PPP without imposing the restrictions, I develop and estimate a more general dynamic structure.

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[1] The early literature dates at least to Cassel (1916). For a survey of early studies see Officer (1976); see also Krugman (1978), Roll (1979), Frenkel (1981), Hakkio (1984), and McCloskey and Zecher (1984).

[2] Studies by Abuaf and Jorion (1990), 15 years, Darby (1983), 7 years of data, Baillie and Selover (1987), 10 years, Meese and Rogoff (1988), 12 years, and Mark (1990), 15 years fail to find evidence of PPP; Cheung and Lai (1993), 16 years, and Oh (1994), 30 years of panel data, on the other hand, find evidence of PPP.

[3] For instance, Abuaf and Jorion (1990), 72 years, find evidence of PPP for 6 of their 8 country pairs, Lothian (1990), 113 years, finds evidence of PPP for 4 of 6 country pairs, and Grilli and Kaminsky (1991), 102 years, find evidence of PPP between the United States and the United Kingdom, which is the only country pair they study. Diebold et al. (1991), 123 years, find evidence of fractional integration, which tends to support PPP, but after removing the fractional differencing term, they also find evidence that unit roots are present in the real exchange rate, which may indicate a failure to find evidence of PPP. Even with fairly long data spans, Adler and Lehmann (1983), 73 years, and Corbae and Ouliaris (1991), 95 years, still find essentially no evidence of PPP.

There is a broader methodological issue underlying the choice between a unit-root test on the real exchange rate and a test based on a more general dynamic structure. As we shall see below, the crucial issue is whether nominal exchange rates and relative price indices are cointegrated with cointegrating vector $(1, -1)$, which would imply the absence of a unit-root in real exchange rates. [4] The test developed here based on the more general dynamic structure can be seen as a *bivariate* test for cointegration between nominal exchange rates and relative price indices. A unit-root test on the real exchange rate is, in effect, a *univariate* test for cointegration between the two variables. Although the univariate test has the virtue of simplicity, it also has costs arising from the dynamic restrictions that are implicitly imposed in collapsing matters to a test on a single variable. My analysis provides a case in which the implicit dynamic restrictions are broadly rejected by the data, so a bivariate test is preferred here.

In Section 2, I show how unit-root tests applied to real exchange rates restrict the dynamic structure relating nominal exchange rates and relative prices. In Section 3, I specify a more general dynamic structure that does not impose these restrictions. I compare results of a univariate unit-root test for PPP with a bivariate test based on the more general dynamic structure in Section 4. I find nearly uniform evidence of PPP.

2. Dynamic structure imposed by unit-root tests

Let s_t and p_t represent, respectively, the natural logarithms of the nominal exchange rate and the relative price of a commodity bundle in period t. Early empirical tests of PPP were typically based on regression of

$$s_t = \alpha_0 + \alpha_1 p_t + \epsilon_t, \tag{2.1}$$

or variants of such regressions, where α_0 and α_1 are parameters and ϵ_t is a period-t error, implicitly assumed to be stationary in these studies. Under PPP, s_t and p_t move one-for-one with each other, so the hypothesis that PPP holds is equivalent to the hypothesis that $\alpha_1 = 1$. [5]

The majority of studies following Meese and Singleton fail to reject a unit root in either s_t or p_t, and take this failure as evidence that a unit root is present in both s_t and p_t. I take the presence of unit roots in s_t and p_t as maintained hypotheses; in section 4 I note that unit roots in s_t and p_t cannot be rejected in the data I consider. For my purposes, the crucial issue is then whether nominal exchange rates and relative price indices are cointegrated with cointegrating vector $(1, -1)$.

[4] Precisely, s_t and p_t are said to be cointegrated with cointegrating vector (δ_1, δ_2) if s_t and p_t each contain a unit root but the linear combination $\delta_1 s_t + \delta_2 p_t$ is stationary.

[5] Under PPP, $\alpha_0 = s_0$, where s_0 is the nominal exchange rate in the base period used to form the price index.

When s_t and p_t are cointegrated with cointegrating vector $(1, -1)$, then the linear combination $\delta_1 s_t + \delta_2 p_t = s_t - p_t$ is stationary. In terms of (2.1), stationarity of $s_t - p_t$ is equivalent to $\alpha_1 = 1$ and stationarity of ϵ_t. Thus PPP is equivalent to cointegration between s_t and p_t with cointegrating vector $(1, -1)$. An *implication* of cointegration with cointegrating vector $(1, -1)$ is that the real exchange rate does not contain a unit root.

Conversely, when s_t and p_t are not cointegrated with cointegrating vector $(1, -1)$, then PPP does not hold and the real exchange rate contains a unit root. Therefore one way to test whether s_t and p_t are cointegrated with cointegrating vector $(1, -1)$ is to test for a unit root in the logarithm of the real exchange rate, $r_t = s_t - p_t$. This is the logic underlying unit-root tests of PPP.

In practice, tests for unit roots in r_t are typically augmented Dickey–Fuller tests based on estimation of

$$\Delta r_t = \beta_0 + \beta_1 r_{t-1} + \sum_{i=1}^{K} \beta_{i+1} \Delta r_{t-i} + \eta_t, \tag{2.2}$$

where Δ is the first difference operator and η_t is a period-t error that is assumed to be serially uncorrelated. If $\beta_1 < 0$, then r_t does not have a unit root so s_t and p_t must be cointegrated with cointegrating vector $(1, -1)$. Thus an estimated value of β_1 that is statistically significantly less than zero is evidence of PPP. If $\beta_1 = 0$, then r_t has a unit root so s_t and p_t cannot be cointegrated with cointegrating vector $(1, -1)$. Thus an estimated value of β_1 that is statistically indistinguishable from zero is failure to find evidence of PPP.

To see the bivariate dynamic structure implicitly used to collapse a bivariate test for cointegration between s_t and p_t with cointegrating vector $(1, -1)$ to a univariate test, note that (2.2) is equivalent to

$$\Delta s_t = \beta_0 + \Delta p_t + \beta_1 (s_{t-1} - p_{t-1}) + \sum_{i=1}^{K} \beta_{i+1} (\Delta s_{t-i} - \Delta p_{t-i}) + \eta_t. \tag{2.2'}$$

Equation (2.2′) can be seen as a restricted error correction model. To understand this, note that $s_{t-1} - p_{t-1}$ is the deviation from PPP in period $t-1$. With $\beta_1 < 0$, the growth rate of the exchange rate in period t is reduced if the exchange rate exceeded the relative price in the previous period. Thus the term $\beta_1(s_{t-1} - p_{t-1})$ provides a "correction" to the growth rate of the exchange rate that depends on the preceding deviation from PPP. If $\beta_1 < 0$ there is a tendency to return to PPP over time, so β_1 can be thought of as capturing the long-run adjustment of the process. If $\beta_1 = 0$, on the other hand, there is no relation between the growth rate of the exchange rate and the preceding deviation from PPP, and hence no tendency to return to PPP.

One important restriction apparent in (2.2′) is that the coefficient on Δp_t equals one by construction. An additional restriction is that the coefficients on lagged values of Δs_t and Δp_t are constrained by (2.2′) to be equal and opposite in sign. These observations stem from work by Kremers et al. (1992), who compare tests for cointegration and note that a Dickey–Fuller test imposes restrictions on an error correction model. As noted, when s_t and p_t have unit roots but are not cointegrated, r_t has a unit root and $\beta_1 = 0$. Testing for a unit root in (2.2) is therefore equivalent to testing for no cointegration in (2.2′) *and also* imposing the restrictions that the coefficient on Δp_t equals one and that the coefficients on lagged values of Δs_t and Δp_t are equal and opposite in sign.

3. A more general dynamic structure

We wish to test the hypothesis that s_t and p_t are cointegrated with cointegrating vector $(1, -1)$ in a generalized dynamic structure that relaxes the restrictions imposed by unit-root tests. In specifying the econometrics, it is also important to take account of the fact that nominal exchange rates and relative price indices are both endogenous variables. This joint endogeneity of s_t and p_t means that estimators of (2.2′) are biased even in an infinite sample. Phillips and Loretan (1991) derive the asymptotic bias and show that inclusion of at least one lead of the differenced regressor (here, the growth rate of the relative price index) can eliminate the bias. When the number of leads included is sufficient to eliminate the bias, then the error term in the dynamic structure is serially uncorrelated and the estimated t-statistic for each coefficient (the estimated coefficient divided by the estimated standard error) is asymptotically normally distributed under the null hypothesis that s_t and p_t are not cointegrated. This provides a way of testing whether the estimated model contains a sufficient number of leads to account satisfactorily for joint endogeneity.

To allow for the joint endogeneity of nominal exchange rates and relative price indices and to relax the dynamic restrictions imposed by unit-root tests, I estimate

$$\Delta s_t = \beta_0 + \gamma \Delta p_t + \beta_1 (s_{t-1} - p_{t-1}) + \beta_2 \Delta p_{t+1} + \sum_{i=1}^{K} (\beta_{pi+2} \Delta p_{t-i} + \beta_{si+2} \Delta s_{t-i}) + \eta_t, \qquad (3.1)$$

which can be seen as a more general error correction model. The dynamic structure in (3.1) generalizes the structure in (2.2′) by including a lead Δp_{t+1} with coefficient β_2, by letting Δp_t have coefficient γ instead of one, and by no longer constraining β_{pi+2} to equal $-\beta_{si+2}$. To determine the number of lags, I start out with the case $K = 1$. If one lag is insufficient, the error in (3.1) will be serially correlated. In Section 4 I run a diagnostic for serial correlation in the residuals. If no evidence of serial correlation is found, then one lag is sufficient to ensure that estimated t-statistics are asymptotically normally distributed.[6] If evidence of serial correlation is found, I add an additional lag of both Δp_t and Δs_t.

As before, failure to reject the hypothesis that $\beta_1 = 0$ is failure to find evidence of PPP.

4. Empirical implementation

I study annual data covering 1927–1990 (64 years). Because I have data on six countries (Canada, France, Germany, Great Britain, Italy, and the United States), I have 15 different country pairs. I use standard consumer price indices as these are widely used in earlier studies; this provides for rough comparability with earlier studies. For 1927–1971, data are from Lee (1976) except for the German consumer price index, which is from *Bevolkerung und Wirtschaft: 1872–1972* (Statisches Bundesamt, 1972). After 1971, data are from *International Financial Statistics* (International Monetary Fund, 1991).

In preliminary work I performed unit-root tests on both s_t and p_t. In accord with earlier studies, I was unable to reject a unit root in either series for each country pair. For the analysis of Section 3 to be valid, both s_t and p_t may have one but not two unit roots. Although there is little evidence that nominal exchange rates contain two unit roots, much attention has been focused on whether prices have two unit roots. Recent analyses of price indices confirm that prices have one unit root and reject the hypothesis that prices have two unit roots. For monthly data Baillie and Pecchenino (1991), 207 months, find evidence of only one unit root for U.S. and U.K. price series, Baillie et al. (1994), 536 months, find evidence of only one unit root for all ten countries they study, and Hassler and Wolters (1995), 276 months, find evidence of only one unit root for all five countries they study.[7] To provide further evidence that the first difference of both s_t and p_t is appropriate for study, I report the first ten sample autocorrelations of the first difference of the logarithm of each real exchange rate series in Table 1.

I compare the results of two standard unit-root tests on the natural logarithm of the real exchange rate (2.2) with the results of a test for cointegration using the generalized dynamics (3.1). To allow for serial correlation and heterogeneity in the residuals of (2.2) I use the test statistic proposed by Phillips and Perron (1988). This test statistic is $T(\hat{\beta}_1 - 1) - \hat{\delta}$, where T is the sample size, $\hat{\beta}_1$ is the ordinary least squares estimator of β_1 in (2.2), K equals zero, and $\hat{\delta}$ is a correction term that renders the distribution of the test statistic invariant to serial correlation or

[6] Because serial correlation in the residuals also occurs when one lead is insufficient, failure to find evidence of serial correlation also indicates that the number of leads is sufficient.

[7] Although Baillie et al. (1994) and Hassler and Wolters (1995) find evidence of long memory in post-war inflation, the longer sample of annual inflation data back to 1927 does not seem to exhibit such persistence and can more reasonably be regarded as an I(0) process.

heterogeneity in the residuals. I also construct the augmented Dickey–Fuller test statistic based on the t-ratio for $\hat{\beta}_1$. To select the number of lagged growth rates for the Dicky–Fuller test (K), I initially set K = 10 and then perform a sequence of regressions, each time reducing K by one, finally stopping when $\hat{\beta}_K$, the ordinary least squares estimator of β_K in (2.2), is statistically distinct from zero.

Results for the standard unit-root tests on the natural logarithm of the real exchange rate are in Table 2. The first column lists the country pairs. The second column contains estimated values of the Phillips–Perron test statistic. The third column contains estimated values of the augmented Dickey–Fuller test statistic

Table 1
Autocorrelation coefficients of lags 1 through 10 of the first-differenced log real exchange rate

Germany/Canada	Germany/France	Germany/Great Britain
0.263	−0.189	0.011
0.122	0.235	−0.034
−0.116	0.062	0.145
−0.050	−0.238	−0.157
−0.057	−0.079	0.133
0.031	−0.128	−0.060
0.166	−0.163	−0.181
0.036	−0.001	−0.027
−0.002	0.109	−0.031
−0.162	−0.133	0.066
Germany/Italy	**Germany/United States**	**United States/Canada**
−0.075	0.257	0.216
0.021	0.058	−0.049
−0.168	−0.008	0.041
−0.154	0.005	−0.021
0.170	−0.022	−0.033
−0.207	−0.060	−0.156
−0.079	0.006	0.054
0.213	−0.018	0.005
0.087	−0.103	−0.301
0.049	−0.087	−0.252
United States/France	**United States/Great**	**United States/Italy**
−0.136	0.062	0.009
0.258	0.078	0.014
−0.144	−0.171	−0.338
−0.213	−0.215	−0.051
−0.201	−0.100	0.062
−0.192	−0.120	−0.139
−0.060	0.012	−0.008
−0.007	0.223	−0.216
0.147	−0.143	0.097
−0.072	0.005	

Table 1 (continued)

Canada/France	Canada/Great Britain	Canada/Italy
−0.145	0.106	0.007
0.257	0.181	0.065
−0.131	−0.248	−0.314
−0.238	−0.256	−0.083
−0.173	−0.125	0.088
−0.181	−0.007	−0.162
−0.044	0.022	−0.029
−0.024	0.160	−0.200
0.121	−0.080	0.079
−0.064	−0.104	0.030
France/Great Britain	**France/Italy**	**Italy/Great Britain**
−0.242	0.005	−0.080
0.244	−0.316	0.002
−0.047	−0.060	−0.343
−0.261	0.069	−0.157
0.047	0.328	0.250
−0.260	−0.073	−0.186
−0.042	−0.281	0.085
0.009	−0.101	−0.259
0.041	−0.038	0.054
−0.031	−0.058	0.091

Table 2
Unit-root test for the log of real exchange rates annual data 1927–1990

Currencies	$T(\hat{\beta}_1 - 1) - \hat{\delta}$	ADF	K
Germany/Canada	−5.79	−2.43	7
Germany/France	−11.76	−2.76	7
Germany/Great Britain	−12.32	−2.83	5
Germany/Italy	−19.85 *	−3.52 *	5
Germany/United States	−6.36	−2.13	1
United States/Canada	−12.06	−4.07 *	7
United States/France	−21.97 *	−4.85 *	2
United States/Great Britain	−10.43	−2.23	8
United States/Italy	−17.62 *	−3.10 *	0
Canada/France	−19.43 *	−4.43 *	2
Canada/Great Britain	−11.32	−2.79	8
Canada/Italy	−15.85 *	−3.22 *	5
France/Great Britain	−19.04 *	−3.88 *	2
France/Italy	−16.08 *	−3.05 *	5
Italy/Great Britain	−22.43 *	−3.96 *	5

* Significant at 5 percent level.

and the fourth column contains the associated value of K. The critical value for a test with a size of five percent is -13.3 for the Phillips–Perron test statistic (from Fuller, 1976, Table 8.5.1) and -2.92 for the augmented Dickey–Fuller test statistic. With the critical value tabulated for a sample size 50, I am able to reject a unit root with a five percent significance level for only 8 of the 15 country pairs for the Phillips–Perron test statistic and for only 9 of the 15 country pairs for the augmented Dickey–Fuller test statistic. Put differently, I fail to find evidence of PPP for nearly half of the country pairs in the sample. These findings are broadly consistent with earlier studies that perform unit-root tests on the real exchange rate using relatively long data spans.

I report estimated coefficients for the more general dynamic structure (3.1) in Table 3. Because $\beta_1 < 0$ when s_t and p_t are cointegrated, the appropriate test is one-sided. Asymptotic critical values can be misleading for moderately sized samples, so I perform monte carlo simulations to obtain more accurate critical values. The simulations consist of 1000 iterations where, for each iteration, I use (3.1) as the data generating process for Δs_t and take $\beta_1 = 0$ because the null hypothesis is that s_t and p_t are not cointegrated; remaining coefficients are representative of estimated values: $\beta_0 = 0.2$; $\gamma = 0.7$; $\beta_2 = 0.6$; $\beta_{p3} = -0.1$; and $\beta_{s3} = 0.2$. The data generating process for Δp_t is a random walk and the errors for the two processes are correlated to generate endogeneity. Specifically, the error for Δp_t equals $0.4u_1 + u_2$ where u_1 and u_2 are independent normal random variables and u_1 is the error in the equation generating Δs_t (results are not sensitive to the choice of coefficients in the error for Δp_t). The critical value for a sample size of 64 is -2.0. (Our results are unchanged if the asymptotic critical value from a normal distribution, -1.65, is used.) For 14 of the 15 country pairs I find evidence of PPP. Specifically, $\hat{\beta}_1$ is (statistically) significantly less than zero for every country pair except Germany/France. For Germany/France, the sign of β_1 is still negative. Thus the more general dynamic structure yields nearly uniform evidence of PPP, in that the estimate of β_1 is negative for each country pair and significant for 14 of 15 country pairs.

We check that the dynamic structure of (3.1) is not misspecified in that it includes enough leads and lags to ensure that the error is serially uncorrelated and hence that statistical inference based on asymptotic normality is valid. [8] Because a lagged dependent variable is included as a regressor in (3.1), an appropriate procedure to test for serial correlation in the residuals is to regress the residuals on

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[8] Edison (1987), Johnson (1990), and Kim (1990) use error correction models to study PPP and find evidence of PPP while Enders (1989) and Thom (1989) use error correction models and fail to find evidence of PPP. These studies do not treat nominal exchange rates and relative price indices as jointly endogenous.

Table 3
Estimation of generalized dynamic structure
$\Delta s_t = \beta_0 + \gamma \Delta p_t + \beta_1 (s_{t-1} - p_{t-1}) + \beta_2 \Delta p_{t+1} + \beta_{p3} \Delta p_{t-1} + \beta_{s3} \Delta s_{t-1} + \eta_t$
Annual data 1927–1990 [a]

Currencies	β_0	γ	β_1	β_2	β_{p3}	β_{s3}
Germany/Canada	0.11 (0.06)	−0.09 (0.34)	−0.10 (0.04)	0.66 (0.34)	0.14 (0.33)	0.47 (0.12)
Germany/France	−0.05 (0.04)	0.93 (0.24)	−0.08 (0.07)	−0.44 (0.21)	0.47 (0.25)	−0.15 (0.14)
Germany/Great Britain	0.36 (0.16)	0.45 (0.26)	−0.17 (0.07)	0.80 (0.24)	−0.23 (0.24)	0.13 (0.13)
Germany/Italy	−0.24 (0.06)	0.62 (0.12)	−0.35 (0.09)	0.20 (0.10)	−0.15 (0.12)	0.02 (0.13)
Germany/United States	0.14 (0.06)	−0.18 (0.32)	−0.12 (0.05)	0.89 (0.31)	0.33 (0.30)	0.51 (0.12)
United States/Canada	−0.01 (0.01)	0.19 (0.25)	−0.19 (0.06)	0.57 (0.23)	−0.26 (0.23)	0.41 (0.13)
United States/France	−0.53 (0.16)	1.68 (0.33)	−0.34 (0.10)	−0.64 (0.26)	−0.04 (0.34)	0.04 (0.13)
United States/Great Britain	0.20 (0.08)	0.73 (0.37)	−0.20 (0.08)	0.51 (0.34)	−0.33 (0.35)	0.20 (0.14)
United States/Italy	−0.91 (0.18)	0.57 (0.12)	−0.50 (0.10)	0.12 (0.11)	−0.35 (0.14)	0.19 (0.11)
Canada/France	−0.46 (0.15)	1.48 (0.37)	−0.29 (0.09)	−0.68 (0.28)	0.08 (0.36)	−0.02 (0.13)
Canada/Great Britain	0.14 (0.08)	0.32 (0.39)	−0.14 (0.07)	0.47 (0.38)	−0.45 (0.37)	0.17 (0.14)
Canada/Italy	−0.83 (0.16)	0.54 (0.13)	−0.46 (0.09)	0.11 (0.11)	−0.31 (0.14)	0.13 (0.11)
France/Great Britain	0.70 (0.24)	1.31 (0.30)	−0.28 (0.09)	−0.39 (0.23)	0.16 (0.29)	−0.12 (0.13)
France/Italy	−0.07 (0.02)	0.90 (0.08)	−0.36 (0.08)	−0.04 (0.08)	−0.32 (0.13)	0.05 (0.12)
Italy/Great Britain	1.74 (0.29)	0.65 (0.11)	−0.63 (0.10)	0.13 (0.09)	−0.32 (0.13)	0.12 (0.11)

[a] Estimated standard errors in parentheses.

Table 4
Testing for serial correlation in the residuals annual data 1927–1990

Currencies	F-Statistic
Germany/Canada	0.56
Germany/France	1.08
Germany/Great Britain	1.55
Germany/Italy	1.77
Germany/United States	0.48
United States/Canada	1.05
United States/France	1.51
United States/Great Britain	0.73
United States/Italy	2.00
Canada/France	1.22
Canada/Great Britain	1.34
Canada/Italy	1.48
France/Great Britain	1.44
France/Italy	2.26
Italy/Great Britain	1.45

all regressors in (3.1) and on lagged values of the residuals, and to test the joint significance of all coefficients. I do this using three lagged residuals. The test statistic is the F-statistic for the null hypothesis that all the coefficients are jointly equal to zero. The critical value for a five percent level of significance is 2.13, so estimated values below 2.13 fail to reject the null hypothesis of no serial correlation in the residuals. Estimated F-statistics are reported in Table 4. For each country pair except France/Italy there is no evidence of residual serial correlation, so for 14 country pairs the dynamic specification appears to include enough leads and lags to permit valid statistical inference.

Because there is evidence of residual serial correlation for France/Italy, I include an additional lag of Δp_t and Δs_t and re-estimate the generalized model for France/Italy. The estimated coefficients are reported in Table 5. The estimated t-statistic for β_1 is less than the critical value, as before. I re-test this dynamic specification for sufficient leads and lags. The critical value for the F-statistic that the coefficients of the equation are jointly equal to zero is 2.0. The estimated F-statistic of 1.2 indicates that no further leads or lags need be included.

The estimated values in Table 3 (for all country pairs except France/Italy) and Table 5 (for France/Italy) allow us to judge the impact of the restrictions imposed by the unit-root test. I use the estimated values to construct a Wald test statistic of the null hypothesis that the restrictions imposed by the unit-root test ($\gamma = 1$, $\beta_2 = 0$, $\beta_{p3} = -\beta_{s3}$) are supported by the data. The value of the test statistic is reported in Table 6. The critical value is 7.81, so estimated values below 7.81 fail to reject the null hypothesis. For 11 of the 15 country pairs, I can reject the

Table 5
Estimation of $\Delta s_t = \beta_o + \gamma \Delta p_t + \beta_1(s_{t-1} - p_{t-1}) + \beta_2 \Delta p_{t+1} + \beta_{p3} \Delta p_{t-1} + \beta_{s3} \Delta s_{t-1} + \beta_{p4} \Delta p_{t-2} + \beta_{s4} \Delta s_{t-2} + \eta_t$
Annual data 1927–1990 [a]

Currency	β_0	γ	β_1	β_2	β_{p3}	β_{s3}	β_{p4}	β_{s4}
France/Italy	−0.04 (0.02)	0.99 (0.09)	−0.23 (0.09)	−0.05 (0.08)	−0.44 (0.13)	0.07 (0.11)	0.31 (0.13)	−0.08 (0.11)

[a] Estimated standard errors in parentheses.

Table 6
Tests of univariate unit-root restrictions annual data 1927–1990

Currencies	Chi-squared statistic
Germany/Canada	14.41 *
Germany/France	7.97 *
Germany/Great Britain	14.14 *
Germany/Italy	10.50 *
Germany/United States	22.48 *
United States/Canada	20.60 *
United States/France	6.85
United States/Great Britain	4.21
United States/Italy	20.35 *
Canada/France	6.14
Canada/Great Britain	8.14 *
Canada/Italy	21.04 *
France/Great Britain	3.79
France/Italy	10.17 [a]*
Italy/Great Britain	15.33 *

* Significant at five percent level.
[a] Because the estimated dynamic structure for France/Italy includes an additional lag of both Δp_t and Δs_t, the test statistic includes the fourth restriction $\beta_{p4} = -\beta_{s4}$. The appropriate critical value is 9.49.

restrictions imposed by the unit-root test. For three of the four country pairs for which I cannot reject the restrictions, the unit-root test provides evidence of PPP.

5. Conclusion

Unit-root tests of purchasing power parity impose restrictions on the dynamic structure relating nominal exchange rates and relative price indices. I specify a more general dynamic structure that relaxes these restrictions and treats nominal exchange rates and relative price indices as jointly endogenous. I then study a fairly long data set that has been employed widely in earlier studies, and I compare unit-root tests with tests based on the generalized dynamic structure. Unit-root tests provide evidence of PPP for only 8 of the 15 country pairs in the study, roughly in line with earlier results. Tests based on the generalized dynamic structure provide evidence of PPP for 14 of the 15 country pairs. To judge which of these test procedures is to be preferred, I perform Wald tests on the restrictions implicit in unit-root tests. For 11 of the 15 country pairs, I reject the restrictions implicit in unit-root tests of PPP. Taken as a whole, the results indicate that even with a fairly long data set, the restrictions implicit in unit-root tests matter. The results also provide strong evidence that purchasing power parity holds across developed countries.

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