

Testing for absolute purchasing power parity

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Purchasing power parity (PPP) is an equilibrium condition equating the nominal exchange rate between two countries with the relative price of an identical bundle of goods in each country. Previous time-series researchers use price indices to study PPP, so they test *relative* PPP. We use new data that measures price levels, so we test *absolute* PPP. Price levels provide a test of absolute PPP because, unlike price indices, price levels do not contain a base period in which the nominal exchange rate equals the price ratio by construction. We find support for absolute PPP. (JEL F31). Copyright © 1996 Elsevier Science Ltd

Purchasing power parity (PPP) is an equilibrium condition equating the nominal exchange rate between two countries with the price ratio of an identical bundle of goods in each country. If the price ratio between the two countries differs from the nominal exchange rate and arbitrage opportunities exist, the resulting trade in goods equates the price ratio with the nominal exchange rate. Because there are costs to trade, the nominal exchange rate rarely equals the price ratio in a given period. Therefore tests for PPP are tests for the tendency of the nominal exchange rate to equal the price ratio. Previous researchers have used price *indices*, so they test for *relative* PPP. We use new data that measures price *levels*, so we test for *absolute* PPP. Relative PPP is a condition

equating changes in the nominal exchange rate to changes in the price ratio. Absolute PPP is a condition equating the level of the nominal exchange rate with the level of the price ratio. Because relative PPP does not imply absolute PPP, previous research supporting relative PPP does not address the validity of absolute PPP.

Because nominal exchange rates and the ratio of prices may contain unit roots, recent research tests for persistence in the deviation of the nominal exchange rate from the price ratio. To do so, researchers test for a unit root in the difference between the nominal exchange rate and the ratio of the price indices.¹ If a unit root is not rejected, there is evidence that arbitrage does not eliminate the difference between the nominal exchange rate and the ratio of price indices, which in turn provides evidence against relative PPP. Earlier researchers who used short spans of data usually found evidence against relative PPP.² Because the power of tests for unit roots increases with the span for the data, more recent researchers who use long spans of data generally find evidence supporting relative PPP.³ Several recent researchers find evidence supporting relative PPP even with short spans of data.⁴

Of course, a finding that the difference between the nominal exchange rate and the ratio of the price indices does not contain a unit root is only a necessary and not a sufficient condition for relative PPP. Given the bulk of evidence rejecting a unit root in this difference, some researchers have taken the additional step of testing the estimated coefficient that measures the relation of changes in the nominal exchange rate to changes in the price ratio. Relative PPP implies that the coefficient equals one. Interestingly, the results are inconclusive.⁵ Unfortunately, several of these researchers have based test statistics on least squares estimators. As is explained in detail below, these test statistics may not be valid. We test the estimated coefficients that measure the relation between the level of nominal exchange rates and the ratio of prices. Because our tests are based on modified least squares estimators, described in more detail below, our test statistics are valid. We find support for absolute PPP for nearly one-third of the country pairs that we study, and we find support for relative PPP for nearly two-thirds of the country pairs that we study.

In the next section we discuss absolute and relative PPP and clarify how we use the terms in our paper. Because our data have not been studied extensively, we describe the data in some detail in Section II. In Section III, we describe our estimators and present results, and Section IV concludes.

I. Absolute and relative PPP

To define symbolically absolute and relative PPP, let the price of a bundle of commodities in country i in period t be P_t^i .⁶ The commodity bundle can contain both traded and non-traded goods, but it is assumed that the weights used to construct the price levels P_t^i and P_t^j are identical for both countries. The nominal exchange rate in period t between country i and country j (that is, the number of units of country i 's currency required to buy one unit of

country j 's currency) is S_t^{ij} . Absolute PPP is

$$\langle 1 \rangle \quad S_t^{ij} = P_t^i / P_t^j.$$

Because there are costs to trade, the nominal exchange rate rarely equals the price ratio in a given period. As a result, econometric tests for absolute PPP are based on estimators of the parameters β_0 and β_1 in

$$\langle 2 \rangle \quad \ln S_t^{ij} = \beta_0 + \beta_1 \ln(P_t^i / P_t^j) + e_t,$$

where e_t is a period- t error that captures time-varying costs to trade. The null hypothesis of absolute PPP is $\beta_0 = 0$ and $\beta_1 = 1$.

Econometric tests of absolute PPP require that price levels be used as the regressor in $\langle 2 \rangle$. If researchers use price indices as the regressor in $\langle 2 \rangle$, they test relative PPP. To understand why, note that if price indices are used as the regressor in $\langle 2 \rangle$, the statement of PPP that underlies such a test is not $\langle 1 \rangle$ but rather

$$\langle 3 \rangle \quad S_t^{ij} / S_B^{ij} = (P_t^i / P_B^i) / (P_t^j / P_B^j),$$

where subscript B denotes the base period of the price index. By construction, $\langle 3 \rangle$ holds for $t = B$. From $\langle 3 \rangle$ it is clear that econometric tests for PPP that use price indices test for the relation between the deviation of the exchange rate from its base period and the deviation of the ratio of price levels from their base periods. Such tests yield information about the value of β_1 in $\langle 2 \rangle$, but they do not yield information about the value of β_0 . As a result, researchers using price indices to estimate $\langle 2 \rangle$ can only test for relative PPP, they cannot test for absolute PPP.

Many researchers test for the proportionality of changes in exchange rates and relative price indices with the first-difference of each of the variables in $\langle 3 \rangle$ to account for the possibility that both exchange rates and price indices contain unit roots.⁷ As is now well known, even if nominal exchange rates and price indices contain unit roots, estimators from a regression with only first-differenced data are inefficient relative to estimators from a regression with data in levels.⁸

To our knowledge, all previous time-series tests for PPP are based on price indices and are tests for relative PPP. Testing for absolute PPP can only be accomplished by testing for the equality between the nominal exchange rate and the ratio of price levels. A comparison of $\langle 1 \rangle$ and $\langle 3 \rangle$ reveals that while absolute PPP implies relative PPP, the reverse does not hold. Therefore previous evidence that provides support for relative PPP ($\beta_1 = 1$) provides only incomplete support for absolute PPP ($\beta_0 = 0$ and $\beta_1 = 1$).⁹

II. Data

Our annual exchange rate and price level data are from *Internationaler Vergleich der Preise für die Lebenshaltung* published by the German Statistical Office (Statistisches Bundesamt). The series runs from 1927 to 1992 for a total of 66 observations. Because a primary purpose of the information is to adjust salaries of German diplomats and foreign service personnel stationed outside Germany, the price levels are cost of living measures primarily for the capital

of each country. The price levels are constructed using weights that reflect the spending pattern of a four person household in the Federal Republic of Germany.¹⁰ For several major countries, price level data are also constructed using weights that reflect the spending pattern of a household in the foreign country. We do not use the data with foreign weights because it does not span the entire sample period for all countries. For post-second World War data, the weights cover an average of 221 goods and services. The German Statistical Office is in the process of improving these data in several ways including increasing the average number of goods to 466. Each price series is not sampled annually. Rather, the German Statistical Office samples the data at intervals and uses a consumer price index to construct the data between sampling dates.

Because we have data on six countries (Canada, France, Germany, Italy, the UK and the USA) we have 15 different country pairs. The German Statistical Office does not publish individual price levels for the various countries. Instead it publishes the price ratio between Germany and each of the other five countries. The price ratio between any two other countries is then the ratio of the German price ratios for those two countries. For example, the price ratio between France and Italy in period t , P_t^F/P_t^I , equals $(P_t^F/P_t^G)/(P_t^I/P_t^G)$.

III. Evidence

To test for absolute PPP we estimate $\beta = (\beta_0, \beta_1)'$ in (2). We work with nominal exchange rates and price levels directly rather than the real exchange rate, which is the nominal exchange rate divided by the ratio of the price levels. We do so because reducing the bivariate relation to a univariate relation imposes restrictions on the short-run behavior of nominal exchange rates and price levels that can reduce the power of univariate statistical tests.¹¹ Specifically, our tests are not tests that absolute PPP holds instantaneously in every time period. Rather, our tests for absolute PPP are tests that deviations from absolute PPP do not persist indefinitely.

Two potential problems arise when working with nominal exchange rates and ratios of price levels. First, unit roots are possibly present in the logarithms of nominal exchange rates and price level ratios. If unit roots are present, then standard asymptotic theory for least squares estimators is invalid. In particular, ordinary least squares (OLS) estimators of the parameters in (2) are asymptotically biased and have a limiting distribution that is not normal and depends on unknown parameters. A second potential problem is that nominal exchange rates are often characterized by more frequent outliers than would be expected if the data are normally distributed. If the data have frequent outliers (that is, are thick tailed), least squares estimators are asymptotically inefficient relative to least absolute deviations (LAD) estimators.

We investigate each of these potential problems in turn. To test for the presence of unit roots in the data, we perform augmented Dickey–Fuller tests and report the results in Table 1. Each row of the table corresponds to the different country pair listed in the first column. Because we have data from six countries, there are 15 country pairs. (Tables 2 and 4 are constructed similarly.)

The second column contains the value of the test statistic for the logarithm of the ratios of price levels and the third column contains the value of the test statistic for the logarithm of the nominal exchange rate. The value of lag listed below each reported test statistic is the number of lagged differences of the series included when constructing the test statistic.¹² Because each of the reported values exceeds the critical value of -2.92 , we cannot reject the presence of a unit root in either series for any country pair at the 5 percent significance level.

To investigate the possibility of thick tails in the distributions for the logarithm of the nominal exchange rate, we estimate the tail-thickness parameter. We calculate numerical estimates of the tail-thickness parameter, denoted α , using a method developed by Hill (1975). Let $\{s_n^{ij}\}_{n=1}^N$ represent the logarithm of the nominal exchange rate, sorted in ascending order, for a particular country pair. We estimate α for each tail of the distribution of s^{ij} with the 10 most extreme observations for that tail. That is, we estimate the left tail with the 10 smallest values of s^{ij} ($n = 1, \dots, n = 10$) and we estimate the right tail with the 10 largest values of s^{ij} ($n = N - 9, \dots, N$). Our estimator of α for the right tail of the distribution of s^{ij} is

$$\langle 4 \rangle \quad \hat{\alpha}_R^{ij} = \left[\sum_{j=1}^{10} (\ln s_{N-j+1}^{ij} - \ln s_{N-10}^{ij}) / 10 \right]^{-1}.$$

To estimate the tail-thickness parameter for the left tail, note that because s^{ij} is the logarithm of the nominal exchange rate, the smallest values are often negative. Thus we estimate α for the left tail of the distribution of s^{ij} as

$$\hat{\alpha}_L^{ij} = \left[\sum_{j=1}^{10} (\ln |s_j^{ij}| - \ln |s_{11}^{ij}|) / 10 \right]^{-1}.$$

Because the value of α indicates the number of finite moments for the distribution of the nominal exchange rate, smaller values of α correspond to thicker tails. A significant number of outliers is reflected in an estimate of α that is smaller than 4. We find that for our data, the smallest value of α is 7.8 for Canada–France, so we find little evidence of thick tails for the logarithm of nominal exchange rates. The finding that annual data on the logarithm of nominal exchange rates do not have thick tails accords with the results in Diebold (1988), who argues that tail thickness is more pronounced as the sampling interval decreases.¹³

In forming test statistics of PPP, it is also important to take account of the fact that nominal exchange rates and the ratios of price levels are both endogenous variables. Because the results in Table 1 indicate the possible presence of a unit root in both the logarithm of the nominal exchange rate and the logarithm of the ratio of price levels, we cannot rely on OLS estimators to form test statistics of PPP. To account for the possible presence of unit roots and the joint endogeneity of nominal exchange rates and the ratios of price levels, we construct fully-modified OLS (FM-OLS) estimators (Phillips and Hansen, 1990).

TABLE 1. Test statistics for a unit root in the logarithm of nominal exchange rates and price level ratios

	Price Level	Nominal Exchange Rate
Canada-France	-1.92260 (lag = 1)	-1.69168 (lag = 2)
Canada-Italy	-1.65267 (lag = 1)	-1.70008 (lag = 2)
Canada-UK	-0.00101 (lag = 1)	-1.08999 (lag = 3)
Germany-Canada	2.23365 (lag = 0)	-0.89937 (lag = 1)
Germany-France	-1.80843 (lag = 1)	-2.06872 (lag = 4)
Germany-Italy	-1.48155 (lag = 1)	-1.94647 (lag = 7)
Germany-UK	0.66876 (lag = 1)	-0.18467 (lag = 1)
Germany-US	1.44464 (lag = 0)	-1.31506 (lag = 1)
France-Italy	-2.35316 (lag = 1)	-1.96404 (lag = 4)
France-UK	-1.96502 (lag = 1)	-1.72856 (lag = 2)
Italy-UK	-1.86304 (lag = 1)	-2.44058 (lag = 7)
US-Canada	-1.43389 (lag = 1)	-2.29473 (lag = 1)
US-France	-1.92462 (lag = 1)	-1.69039 (lag = 2)
US-Italy	-1.59421 (lag = 1)	-1.63433 (lag = 2)
US-UK	0.07066 (lag = 1)	-1.12853 (lag = 2)

Critical value for a test with size 5 percent is -2.92. Lags are the number of added lagged differenced terms.

To describe the FM-OLS estimators, we drop the notation signifying a specific country pair. Let s_t be the period- t logarithm of the nominal exchange rate and x_t^* be the period- t logarithm of the ratio of price levels, both for a generic country pair. Then (2) is equivalent to

$$(5) \quad s_t = x_t \beta + e_t,$$

where $x_t = [1, x_t^*]$. The possible presence of unit roots is introduced through the auxiliary equation

$$(6) \quad x_t^* = x_{t-1}^* + u_t,$$

where u_t is a stationary period- t error. Joint endogeneity of nominal exchange rates and relative price levels is introduced through correlation between e_t and u_t . Let (e_t, u_t) be a bivariate normal random variable with mean zero and non-singular covariance matrix.¹⁴ Because (e_t, u_t) is a bivariate normal random variable

$$\langle 7 \rangle \quad E[s_t | \Delta x_t^*] = x_t \beta + \Delta x_t^* \omega_{21} / \omega_{22},$$

where $\Delta x_t^* = x_t^* - x_{t-1}^*$, and the long-run covariances are $\omega_{21} = \sum_{k=-\infty}^{\infty} E[\Delta x_0^* e_k]$ and $\omega_{22} = \sum_{k=-\infty}^{\infty} E[\Delta x_0^* \Delta x_k^*]$.¹⁵ Note that if e_t and u_t are uncorrelated, that is nominal exchange rates and the ratio of price levels are not jointly endogenous, ω_{21} equals zero and the second term in $\langle 7 \rangle$ vanishes. If nominal exchange rates and the ratio of price levels are jointly endogenous, the correct specification of $\langle 5 \rangle$ is

$$\langle 5' \rangle \quad s_t = x_t \beta + \Delta x_t^* \omega_{21} / \omega_{22} + e_t.$$

Estimating β from $\langle 5' \rangle$, corrects for joint endogeneity, but does not eliminate all nuisance parameters from the limiting distribution of the estimator. To see this, the endogeneity corrected estimator of β from $\langle 5' \rangle$, denoted $\tilde{\beta}$, is

$$\langle 8 \rangle \quad \tilde{\beta} = (x'x)^{-1} x'(s - \Delta x^* \hat{\omega}_{21} / \hat{\omega}_{22}),$$

which equals

$$\langle 9 \rangle \quad \tilde{\beta} = \beta + (x'x)^{-1} x'(e - u \hat{\omega}_{21} / \hat{\omega}_{22}),$$

where $s = [s_1, \dots, s_T]'$, $x = [x_1', \dots, x_T']'$, $\Delta x^* = [\Delta x_1^*, \dots, \Delta x_T^*]'$, $e = [e_1, \dots, e_T]'$, and $u = [u_1, \dots, u_T]'$. If unit roots are present the second and third terms on the right-hand side of $\langle 9 \rangle$ do not vanish asymptotically. In fact, as Phillips and Durlauf (1986) note:

$$\begin{aligned} \langle 10 \rangle \quad T^{-1} x' e &\Rightarrow K_1 + \delta_{21}, \\ T^{-1} x' u &\Rightarrow K_2 + \delta_{22}, \\ T^{-2} x' x &\Rightarrow K_2, \end{aligned}$$

where K_1 and K_2 are functions that depend on the long-run covariances ω_{21} and ω_{22} , respectively; \Rightarrow denotes weak functional convergence; and $\delta_{21} = \sum_{k=0}^{\infty} E[\Delta x_0^* e_k]$ and $\delta_{22} = \sum_{k=0}^{\infty} E[\Delta x_0^* \Delta x_k^*]$.

In forming the correction for the second and third terms on the right-hand side of $\langle 9 \rangle$ we must take care to distinguish between β_0 and β_1 . Because a column vector of 1's does not contain a unit root, the estimator $\tilde{\beta}_0$ does not need to be modified to account for the possible presence of a unit root. Given $\langle 10 \rangle$, straightforward calculations reveal that the appropriate correction for $\tilde{\beta}_1$ is $-T(\hat{\delta}_{21} - \hat{\delta}_{22} \hat{\omega}_{21} / \hat{\omega}_{22})$, so the FM-OLS estimator is

$$\langle 11 \rangle \quad \hat{\beta}_{\text{FM-OLS}} = (x'x)^{-1} \left\{ x'(s - \Delta x^* \hat{\omega}_{21} / \hat{\omega}_{22}) - T[0, 1]' \left(\hat{\delta}_{21} - \hat{\delta}_{22} \hat{\omega}_{21} / \hat{\omega}_{22} \right) \right\},$$

where the vector $[0, 1]'$ ensures that the unit-root correction applies only to the estimator of β_1 .

To form the FM-OLS estimators $\langle 11 \rangle$, we need consistent estimators of $[\delta_{21}, \delta_{22}, \omega_{21}, \omega_{22}]$. The estimator of the long-run covariance δ_{21} is

$$\langle 12 \rangle \quad \hat{\delta}_{21} = T^{-1} \sum_{j=0}^{T-1} w(j) \left[\sum_{t=1}^{T-j} \Delta x_{t+j}^* \hat{e}_t \right],$$

where \hat{e}_t is the period- t residual from the OLS estimates and $w(j)$ is a (kernel) weighting function. The weighting function that we use is the quadratic spectral kernel

$$w(j) = \frac{25}{12\pi^2 j^2} \left[\frac{\sin(6\pi j/5)}{6\pi j/5} - \cos(6\pi j/5) \right],$$

which is asymptotically optimal (Andrews, 1991).¹⁶ To ensure that $\langle 12 \rangle$ is a consistent estimator of δ_{21} , we cannot assign non-zero weights to all of the terms in $\langle 12 \rangle$. To select the correct number of terms we regress \hat{e} on Δx^* and use the estimated coefficient to determine the number of lags from Table 1 in Andrews (1991). (Results are similar if Δx^* is regressed on \hat{e}). The number of included lags differs across country pairs depending on the magnitude of the estimated coefficient. Across the 15 country pairs, the number of included lags ranges from 2 to 20. The estimator of δ_{22} is constructed as in $\langle 12 \rangle$ with Δx_t^* in place of \hat{e}_t . The estimator of ω_{21} is

$$\langle 13 \rangle \quad \hat{\omega}_{21} = T^{-1} \sum_{j=-T+1}^{T-1} w(j) \sum_{t=s_1}^{s_2} \Delta x_{t+j}^* \hat{e}_t,$$

where $(s_1 = -j + 1, s_2 = T)$ if $j < 0$ and $(s_1 = 1, s_2 = T - j)$ if $j \geq 0$. The estimator of ω_{22} is constructed as in $\langle 13 \rangle$ with Δx_t^* in place of \hat{e}_t .

The covariance matrix for the FM-OLS estimator is

$$\hat{\omega}_{11.2} (x'x)^{-1},$$

where the long-run conditional variance of y given Δx^* is $\hat{\omega}_{11.2} = \hat{\omega}_{11} - \hat{\omega}_{21}^2 / \hat{\omega}_{22}$ with $\hat{\omega}_{11}$ formed as in $\langle 13 \rangle$ with \hat{e}_{t+j} in place of Δx_{t+j}^* .

Inference on $\hat{\beta}_{\text{FM-OLS}}$ is easily performed. To test the null hypothesis $R\beta = r$, the Wald-statistic is

$$\langle 14 \rangle \quad (R\hat{\beta}_{\text{FM-OLS}} - r)' [x'x] (R\hat{\beta}_{\text{FM-OLS}} - r) / \hat{\omega}_{11.2} \xrightarrow{d} \chi_q^2,$$

where q is the number of restrictions imposed by the null hypothesis.

Our test of absolute PPP has two components. First, we test for a unit root in the residuals from the FM-OLS estimate of β . The logic is that if absolute PPP holds, there is an equilibrium relation between nominal exchange rates and the ratio of price levels. Because evidence of a unit root in the FM-OLS residuals is evidence against such an equilibrium relation, evidence of a unit root in the FM-OLS residuals is evidence against absolute PPP. The second component of our test is based directly on estimates of the parameters in $\langle 2 \rangle$. Specifically, for the country pairs for which we reject a unit root in the FM-OLS residuals, we then test the joint null hypothesis that $\beta_0 = 0$ and $\beta_1 = 1$.

TABLE 2. Test statistics for a unit root in the fully-modified residuals

	FM Residuals
Canada-France	-3.85 (lag = 2)
Canada-Italy	-3.63 (lag = 2)
Canada-UK	-2.47 (lag = 1)
Germany-Canada	-2.65 (lag = 1)
Germany-France	-4.45 (lag = 7)
Germany-Italy	-3.27 (lag = 2)
Germany-UK	-2.87 (lag = 9)
Germany-US	-2.39 (lag = 1)
France-Italy	-2.55 (lag = 0)
France-UK	-4.37 (lag = 3)
Italy-UK	-3.20 (lag = 2)
US-Canada	-3.25 (lag = 1)
US-France	-3.86 (lag = 2)
US-Italy	-3.75 (lag = 2)
US-UK	-3.08 (lag = 1)

See note to Table 1.

Table 2 contains the results of an augmented Dickey-Fuller test for a unit root in the FM-OLS residuals. The second column in Table 2 contains the estimate of the test statistic for the FM-OLS residuals with the number of lagged differences of the residual series that are included listed in parentheses below each estimated test statistic. For 5 or the 15 country pairs the estimated test statistic exceeds the critical value of -2.92 , so we cannot reject a unit root in the residual series. For the other ten country pairs the estimated test statistic is less than the critical value, so for these 10 country pairs we reject a unit root in the residual series. We conclude that for 10 of the 15 country pairs there is an equilibrium relation between nominal exchange rates and the ratio of price levels.

TABLE 3. OLS and FM-OLS estimates of β_0 and β_1

Country pair	OLS		FM-OLS	
	$\hat{\beta}_0$	$\hat{\beta}_1$	$\hat{\beta}_0$	$\hat{\beta}_1$
Canada–France	–0.302 (0.029)	0.936 (0.021)	–0.313 (0.056)	0.938 (0.041)
Canada–Italy	–0.268 (0.037)	0.986 (0.020)	–0.355 (0.050)	0.971 (0.028)
Canada–UK	0.148 (0.071)	0.663 (0.047)	0.177 (0.186)	0.653 (0.123)
Germany–Canada	–0.026 (0.123)	1.312 (0.148)	0.183 (0.282)	1.107 (0.342)
Germany–France	–0.019 (0.022)	0.918 (0.015)	–0.018 (0.041)	0.916 (0.028)
Germany–Italy	–0.031 (0.028)	0.969 (0.016)	–0.063 (0.056)	0.970 (0.032)
Germany–UK	0.167 (0.065)	0.873 (0.028)	0.344 (0.129)	0.820 (0.055)
Germany–US	–0.130 (0.185)	1.377 (0.204)	–0.068 (0.464)	1.298 (0.514)
France–Italy	0.049 (0.030)	1.099 (0.057)	0.053 (0.068)	1.150 (0.128)
France–UK	0.036 (0.043)	0.945 (0.020)	0.034 (0.074)	0.948 (0.035)
Italy–UK	–0.083 (0.055)	1.000 (0.022)	–0.066 (0.108)	1.006 (0.043)
US–Canada	0.006 (0.012)	0.883 (0.098)	0.011 (0.022)	0.960 (0.177)
US–France	–0.292 (0.027)	0.933 (0.019)	–0.294 (0.051)	0.935 (0.035)
US–Italy	–0.252 (0.033)	0.986 (0.018)	–0.320 (0.045)	0.982 (0.025)
US–UK	–0.094 (0.052)	0.692 (0.036)	0.119 (0.129)	0.681 (0.089)

Standard errors in parentheses.

Table 3 contains estimates of β_0 and β_1 for each country pair. The first two columns contain the OLS estimates, and the last two column contain the FM-OLS estimates of β_0 and β_1 . The standard error of the estimate is presented below each estimate. Because the OLS estimators are asymptotically biased with a non-standard limiting distribution, standard statistical inference is not valid for the OLS estimators. The FM-OLS estimators, in contrast, are asymptotically unbiased with a normal limiting distribution, so standard statistical inference is valid for the FM-OLS estimators. Thus we construct our test statistics from the FM-OLS estimators. For the null hypothesis of relative PPP, that is $\beta_1 = 1$, we find strong support. For 8 of the 10 country pairs for which

TABLE 4. Joint hypothesis test $\beta_0 = 0$ $\beta_1 = 1$

Country pair	Test statistic
Canada–France	31.05
Canada–Italy	55.00
Canada–UK	44.53
Germany–Canada	14.27
Germany–France	10.64
Germany–Italy	2.23
Germany–UK	13.74
Germany–US	7.55
France–Italy	1.38
France–UK	5.51
Italy–UK	0.90
US–Canada	1.12
US–France	33.71
US–Italy	56.21
US–UK	62.08

Critical value for $\chi^2(2)$ at the 5 percent level is 5.99.

we reject a unit root in the FM-OLS residuals, we are unable to reject the univariate null hypothesis that $\beta_1 = 1$.

The finite sample implications of including the modification terms are revealed in a comparison of the OLS and FM-OLS estimates and the estimated standard errors. The point estimates of β_1 are largely unchanged, although for both Germany–Canada and USA–Canada the FM-OLS estimates of β_1 are substantially closer to 1 than are the OLS estimates. The point estimates of β_0 , however, are often changed substantially. In general, the FM-OLS estimates of β_0 are further from zero than are the OLS estimates of β_0 , indicating that the absolute value of the OLS estimator of β_0 tends to be downward biased. Unsurprisingly, the reported OLS standard errors are smaller than the FM-OLS reported standard errors, reflecting the fact that the estimator of the OLS standard error does not account for the non-standard limiting distribution of the OLS estimator.

We report the results for a hypothesis test of absolute PPP in Table 4. The test statistic is constructed as in (14). Because we are testing a set of 2 restrictions, the asymptotic distribution of our test statistic is $\chi^2(2)$. The critical value for a test with a 5 percent significance level is 5.99. For 4 of the 10 country pairs for which we are unable to reject a unit root in the FM-OLS residuals, we are unable to reject the joint null hypothesis that $\beta_0 = 0$ and $\beta_1 = 1$. That is for 4 of the 8 country pairs for which we are unable to reject relative PPP we are also unable to reject absolute PPP. For France–Italy we cannot reject the joint null hypothesis that $\beta_0 = 0$ and $\beta_1 = 1$, but we also cannot reject a unit root in the FM-OLS residuals. This result may be due to

the low power of unit-root tests relative to Wald tests based on FM-OLS estimators. In fact, the evidence against a unit root in the deviations of nominal exchange rates from relative prices is quite strong, as Lothian and Taylor (1996) find with 200 years of data. We also note that in each case for which we cannot reject absolute PPP, both countries are on the same side of the Atlantic ocean, suggesting that transaction costs may play a role in the violations of absolute PPP. Our results are more supportive of absolute PPP than one might expect given the general rejection of the theory.

IV. Conclusions

We use previously unexamined data on price levels to test absolute PPP. With 66 years of data we find support for PPP. In particular, for 8 of the 15 country pairs we cannot reject relative PPP and for 4 of those 8 country pairs we cannot reject absolute PPP. Our results suggest that absolute PPP may indeed hold, in the sense that deviations from absolute PPP do not persist indefinitely. Our findings contrast with the out-of-hand rejection presented in Salvatore (1993, p. 493), who states that the presence of non-traded goods as well as 'transportation costs or other obstructions to the free flow of international trade' imply that 'the absolute PPP theory must be rejected'. Our findings suggest that absolute PPP should be treated as a serious empirical relation that deserves study.

Notes

1. Unit-root tests are time-series tests because they measure persistence through time. Cross-section tests cannot measure persistence through time and so do not address the question of persistence in the deviation of the nominal exchange rate from the price ratio.
2. Studies by Meese and Rogoff (1988, 12 years of data) and Mark (1990, 15 years) fail to reject unit roots.
3. For instance, Abuaf and Jorion (1990, 72 years) find evidence of relative PPP for 6 of their 8 country pairs. Grilli and Kaminsky (1991, 102 years) find evidence of relative PPP between the USA and the UK which is the only country pair they study, Lothian and Taylor (1996, 200 years) find evidence of relative PPP for both of their country pairs, and Steigerwald (1994, 64 years) finds evidence of relative PPP for 14 of the 15 country pairs he studies. Diebold *et al.* (1991, 123 years) find evidence of fractional integration, which tends to support PPP, but after removing the fractional differencing term, they also find evidence that unit roots are present in the real exchange rate, which may indicate a failure to find evidence of PPP. An exception is Corbae and Ouliaris (1991, 95 years) who fail to find evidence of relative PPP between Australia and her major trading partners.
4. For example, Cheung and Lai (1993, 16 years), Pippenger (1993, 16 years), and Frankel and Rose (1995, 22 years of data).
5. Edison (1985) and Cheung and Lai (1993) fail to find evidence that the coefficient equals one while Hakkio (1984) and Edison and Klovland (1987) find evidence that the coefficient equals one. Apte *et al.* (1994) construct instrumental variable estimators and find that the coefficient equals one for one instrument, but does not equal one for other instruments.
6. An alternative definition of PPP uses GDP deflators in place of the price of a bundle of commodities. We follow Dornbusch (1987) in using the price of a bundle of goods, so that PPP is implied by the law of one price.

7. Our use of absolute and relative PPP differ from the usage in several other papers in which absolute PPP refers to $\langle 3 \rangle$ and relative PPP refers to the first-difference of $\langle 3 \rangle$. Because both $\langle 3 \rangle$ and the first-difference of $\langle 3 \rangle$ describe the relation between changes in exchange rates and changes in relative price levels, both define relative PPP.
8. If ratios of price levels contain a unit root and absolute PPP is true, then a regression using only first differences of the nominal exchange rate and first differences of the ratio of price levels is equivalent to a regression using data in levels only if an infinite number of lags are included.
9. Kravis and Lipsey (1983) and Ward (1985) use cross-section data to test for absolute PPP.
10. For the period prior to 1974, the weights correspond to a four person household with median income and included rent. After 1974, the weights correspond to the German consumer price index with rent excluded.
11. Steigerwald (1996) shows that tests based on the bivariate relation provide stronger support for relative PPP than do univariate tests.
12. To select the number of lags we begin with 10 lags in each case and construct the test statistic. If the coefficient on lag 10 is not significant, we reduce the number of lags by one and again construct the test statistic. We repeat the procedure until the estimated coefficient on the largest lag is significant.
13. To reinforce our finding that thick tails are not a problem, we also constructed (fully-modified) least absolute deviations estimates of β_0 and β_1 , which are robust to thick-tailed distributions. The results are similar to those reported in Table 2 through Table 4.
14. The results that follow do not depend on normality of $[e_t, u_t]$ as Phillips (1987) shows. The FM-OLS estimator is valid for non-normal errors with weak restrictions on dependence and heteroskedasticity.
15. The presence of unit roots requires the use of long-run covariance estimators instead of contemporaneous covariance estimators.
16. Finite sample simulations indicate that the quadratic spectral kernel provides the most accurate estimator of long-run covariances.

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